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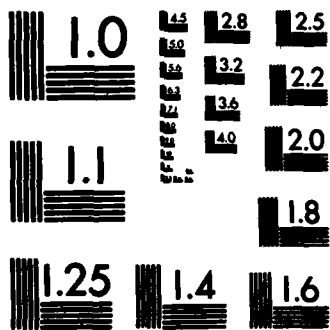
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DISTRIBUTED WIENER-POISSON CONTROL

By

Howard Weiner

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Herbert Solomon, Project Director

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Distributed Wiener-Poisson Control

by Howard Weiner

University of California, Davis
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1. Introduction. Let $W(t)$, $t \geq 0$, $W(0) = 0$ be a standard Wiener process, independent of $N(t)$, $t \geq 0$, $N(0) = 0$ a Poisson process with (constant) unit jumps, and $EN(t) = \lambda t$, $t \geq 0$. Let their sigma fields be $F(t) = \sigma(W(s), 0 \leq s \leq t)$ and $G(t) = \sigma(N(s), 0 \leq s \leq t)$. Let $g(y)$ be a function and $L(g(y))$ a differential operator on g , for example,

$$L(g(y)) = a(y) \frac{\partial^2}{\partial y^2} g(y) + d(y) \frac{\partial}{\partial y} g(y) + c(y)g(y).$$
Let $X(t,y)$ be a stochastic process depending on $t \geq 0$ and a parameter y , such that (for $\frac{\partial X}{\partial t} = X_t$),

$$(1.1) \quad X_t(t,y) = L(X(t,y)) + u(X(t,y),y) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt}$$

$$X(0,y) = g(y) = x.$$

with $\frac{dW(t)}{dt}$ white noise, $\frac{dN(t)}{dt}$ incremental Poisson jump process and where $u(X(t,y),y)$ is measurable with respect to $\sigma(F(t) \cup G(t))$ (i.e. u is non-anticipative) and satisfies, for A a constant, and $B > 0$ a constant,

$$(1.2) \quad |u-A| \leq B,$$

all $0 \leq t \leq T$, $0 < T \leq \infty$ a constant. The cost function for a given u satisfying (1.2) is, for $\alpha > 0$ a constant,

$$(1.3) \quad J(u,y) = \int_0^T e^{-\alpha s} E(X^2(u,s))ds.$$

The object is to characterize the optimal u for which J is minimized. The cases $T < \infty$ and $T = \infty$ are treated separately. The method employs a suitable Bellman equation, a maximum principle for parabolic partial differential-difference equations and the Ito rule. The method follows [4].

2. Finite Interval Control.

Let $T < \infty$. Define, for $0 \leq t \leq T$,

$$(2.1) \quad V = V(x, t, y) = \inf_{|u-A| \leq B} \int_0^t e^{-\alpha s} E(X^2(s, y)) ds$$

and $X(0, y) = g(y) = x$.

By writing $\int_0^t = \int_0^h + \int_h^{t+h} - \int_t^{t+h}$, heuristic arguments ([2], pp. 179-180) yield a Bellman equation

$$(2.2) \quad x^2 + \inf_{|u-A| \leq B} (uV_x) + L(g(y))V_x + \frac{1}{2} y_{xx} - \alpha V - V_t + \lambda(V(x+1, t, y) - V(x, t, y)) = 0,$$

with $u = u(x)$.

On heuristic grounds, a solution to (2.2) is sought such that

$$(2.3) \quad x^2 + [(A-B) + L(g(y))]V_x + \frac{1}{2} V_{xx} - \alpha V - V_t + \lambda(V(x+1, t, y) - V(x, t, y)) = 0$$

for $V_x > 0$, $x > b(t, y)$

and

$$(2.4) \quad x^2 + [(A+B) + L(g(y))]V_x + \frac{1}{2} V_{xx} - \alpha V - V_t + \lambda(V(x+1, t, y) - V(x, t, y)) = 0$$

for $V_x < 0$, $x < b(t, y)$

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for $V_x < 0$, $x < b(t, y)$

where $b(t,y)$ is obtained from the following conditions, letting

$V = V_1$ in (2.3) and $V = V_2$ in (2.4) for $0 \leq t \leq T$, all y :

$$V_1(b(t,y),t,y) = V_2(b(t,y),t,y)$$

$$V_{1,x}(b(t,y),t,y) = V_{2,x}(b(t,y),t,y) = 0$$

$$(2.5) \quad V_1(x,0,y) = V_2(x,0,y) = 0.$$

$$(2.5a) \quad V_{1,xx}(b(t,y),t,y) \geq 0.$$

For R constant, denote

$$(2.6) \quad J(x,t,y,R) = \int_0^t e^{-\alpha s} E((R+L(g(y)))s + W(s) + N(s) + x)^2 ds.$$

A direct computation verifies that $J(x,t,y,A-B)$ is a particular solution to (2.3) and $J(x,t,y,A+B)$ is a particular solution to (2.4).

Assumption 1. There is a non-zero solution $H_1(x,t,y)$ with $H_1(x,0,y) = 0$

to

$$-\alpha H + (A-B + L(g(y)))H_x + \frac{1}{2} H_{xx} - H_t + \lambda(H(x+1,t,y) - H(x,t,y)) = 0$$

such that

$$H_1(x,t,y) = O(e^{-\beta x})$$

$$(2.7) \quad H_{1,xx}(x,t,y) = O(e^{-\delta x})$$

for some $\beta > 0$, $\delta > 0$, all t,y , as $x \rightarrow \infty$.

Also, there is a non-zero solution $H_2(x,t,y)$ with $H_2(x,0,y) = 0$,

to

$$\begin{aligned} -\alpha H + (A+B + L(g(y)))H_x + \frac{1}{2} H_{xx} - H_t \\ + \lambda(H(x+1,t,y) - H(x,t,y)) = 0 \end{aligned}$$

such that

$$H_2(x,t,y) = O(e^{\delta x})$$

$$(2.8) \quad H_{2,xx}(x,t,y) = O(e^{\lambda x})$$

for some $\delta > 0$, $\lambda > 0$, all t, y , as $x \rightarrow -\infty$.

One may then write

$$(2.9) \quad V_1(x,t,y) = J(x,t,y, A-B) + H_1(x,t,y)$$

$$(2.10) \quad V_2(x,t,y) = J(x,t,y, A+B) + H_2(x,t,y)$$

and assume $b(t,y)$ is determined from (2.5), (2.9), (2.10).

This motivates

Theorem 1 Assume the conditions and results of sections 1,2 hold for $0 < T < \infty$.

The optimal u_0 may be expressed as

$$(2.11) \quad u_0(X_0(t,y), y) = \begin{cases} A-B & \text{if } X_0(t,y) > b(T-t,y) \\ A+B & \text{if } X_0(t,y) \leq b(T-t,y) \end{cases}$$

where $b(t,y)$ is obtained from (2.5), (2.9)-(2.10) and

$$(2.12) \quad X_{0t}(t,y) = L(X_0(t,y)) + u_0(X_0(t,y), y) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt}$$

with $X_0(0,y) = g(y) = x$.

Proof Let $D = V_{xx}$.

From (2.3), (2.4) (omitting (t,y) arguments)

$$(2.13) \quad (A+B + L(g(y))) D_x + \frac{1}{2} D_{xx} - uD - D_t - \lambda D = -2 - \lambda D(x+1)$$

where the argument on the left side of (2.13) is x .

From (2.6),

$$(2.14) \quad J_{xx}(x,t,y, A+B) > 0.$$

Assumption 1, (2.7)-(2.10), (2.14), imply that

$$(2.15) \quad V_{1,xx}(x,t,y) > 0, \text{ for each } t,y, \text{ as } x \rightarrow \infty$$

and

$$(2.16) \quad V_{2,xx}(x,t,y) > 0, \text{ for each } t,y, \text{ as } x \rightarrow -\infty.$$

For fixed (t,y) , (omitting the fixed arguments (t,y)), suppose there existed a finite number $r > b$, and a number β , $0 < \beta < 1$ and such that using (2.5a),

$$(2.17) \quad D(x) < 0, \quad b < r - \beta < x < r, \quad D(r) = 0,$$

and

$$(2.17) \quad D(x) > 0, \quad x > r,$$

using (2.15). It follows that for $x \geq r - \beta$, the left side of (2.13) is negative. By ([1], Lemma 1, p.34), D cannot have a negative minimum for $x \geq r - \beta$, a contradiction to (2.17), since (2.15) and (2.5a) hold. Hence $D \geq 0$ for $x \geq b$.

For $b-1 \leq x < b$, a similar argument using (2.16) and that $D(x) > 0$ for $b \leq x < b+1$ yields that $D(x) \geq 0$ for $b-1 \leq x < b$. Continuing by iteration, $D(x) > 0$ for $x < b$, so that $D \geq 0$ for all (x,t,y) .

Also, $V_x(b(t)) = 0$ by (2.5), implying that (2.3), (2.4) (2.9), (2.10) is a solution to the Bellman equation (2.2). To show that u_0 is optimal, define

$$(2.19) \quad K(X(t,y),t,y) = V(X(t,y),T-t,y)e^{-\alpha t}$$

for $0 \leq t \leq T$.

Noting that $K(X(0,y),0,y) = V(x,T,y)$ and $K(X(T,y),T,y) = 0$, the Ito rule yields that ([2], pp. 125-126) for a u and corresponding $X(t,y)$,

$$\begin{aligned}
(2.20) \quad & \int_0^T e^{-\alpha s} X^2(s, y) ds - V(x, T, y) = \\
& \int_0^T e^{-\alpha s} (-\alpha V(X(s, y), T-s, y) - V_t(X(s, y), T-s, y) \\
& \quad + \inf_{|u-A| \leq B} (u(X(s, y), y) V_x(X(s, y), T-s, y)) \\
& \quad + \frac{1}{2} V_{xx}(X(s, y), T-s, y) + L(g(y)) V_x(X(s, y), T-s, y) + X^2(s, y)) ds \\
& \quad + \int_0^T e^{-\alpha s} (V(X(s, y), T-s, y) dN(s) \\
& \quad + \int_0^T e^{-\alpha s} V_x(X(s, y), T-s, y) dW(s) \\
& \quad + \int_0^T e^{-\alpha s} (u(X(s, y), y) V_x(X(s, y), T-s, y) - \inf_{|u-A| \leq B} (u(X(s, y), y) V_x(X(s, y), T-s, y))) ds
\end{aligned}$$

The fourth integral on the right of (2.20) is non-negative. Upon taking expectations of (2.20), the third integral on the right is zero, and one obtains (omitting the arguments from the first and fourth integrals, and combining the first and second integrals, all on the right)

$$\begin{aligned}
(2.21) \quad & \int_0^T e^{-\alpha s} E(X^2(s, y)) ds - V(x, T, y) = \\
& E \int_0^T e^{-\alpha s} (-\alpha V - V_t + \inf_{|u-A| \leq B} (u V_x) + L(g(y)) V_x + \frac{1}{2} V_{xx} + X^2 \\
& \quad + \lambda(V(X(s)) + 1 - V) ds \\
& \quad + E \int_0^T e^{-\alpha s} (u V_x - \inf_{|u-A| \leq B} (u V_x)) ds.
\end{aligned}$$

The first integral on the right of (2.21) is zero by (2.2)-(2.4), and

the second integral is non-negative, and is zero for $u = u_0$, $X = X_0$.

Hence (2.21) yields that

$$(2.22) \quad \int_0^T e^{-\alpha s} E(X^2(s, y)) ds \geq V(x, T, y)$$

and

$$(2.23) \quad \int_0^T e^{-\alpha s} E(X_0^2(s, y)) ds = V(x, T, y),$$

and (2.22), (2.23) imply u_0 is optimal.

3. Infinite Interval Control

Assume the conditions of section 1 hold for $T = \infty$ so that (1.3) is now

$$(3.1) \quad J(u, y) = \int_0^\infty e^{-\alpha s} E(X^2(s, y)) ds.$$

Define

$$(3.2) \quad V = V(x, y) = \inf_{|u-A| \leq B} \int_0^\infty e^{-\alpha s} E(X^2(s, y)) ds$$

with $X(0, y) = g(y) = x$.

By writing $\int_0^\infty = \int_0^h + \int_h^\infty$, heuristic arguments ([2], pp. 179-180) yield a Bellman equation

$$(3.3) \quad x^2 - \alpha V(x, y) + \frac{1}{2} V_{xx}(x, y) + \inf_{|u-A| \leq B} (u(x, y) V_x(x, y)) \\ + L(g(y)) V_x(x, y) + \lambda (V(x+1, y) - V(x, y)) = 0.$$

On heuristic grounds, a solution to (3.3) is sought such that (omitting the (x, y) arguments)

$$(3.4) \quad x^2 - \alpha V + \frac{1}{2} V_{xx} + (A-B + L(g(y)))V_x + \lambda(V(x+1,y)-V) = 0$$

$$\text{for } V_x > 0, x > b(y)$$

and

$$(3.5) \quad x^2 - \alpha V + \frac{1}{2} V_{xx} + (A+B+L(g(y)))V_x + \lambda(V(x+1,y)-V) = 0$$

$$\text{for } V_x < 0, x < b(y),$$

where $b(y)$ is obtained from the following matching conditions, letting $V = V_1$ in (3.4) and $V = V_2$ in (3.5):

$$V_1(b(y), y) = V_2(b(y), y)$$

$$(3.6) \quad V_{1,x}(b(y), y) = V_{2,x}(b(y), y) = 0$$

$$(3.6a) \quad V_{1,xx}(b(y), y) \geq 0.$$

For R constant, denote

$$(3.7) \quad J(x, y, R) = \int_0^\infty e^{-\alpha s} E((R+L(g(y)))s + W(s) + N(s) + \pi)^2 ds.$$

A computation verifies that $J(x, y, A-B)$ is a particular solution to (3.4) and $J(x, y, A+B)$ is a particular solution to (3.5).

Assumption 2. There is a non-zero solution $H_1(x, y)$ to (omitting (x, y) argument)

$$-\alpha H + (A-B + L(g(y)))H_x + \frac{1}{2} H_{xx} + \lambda(H(x+1, y) - H) = 0$$

such that

$$H_1(x, y) = O(e^{-\beta x})$$

$$(3.8) \quad H_{1,xx}(x, y) = O(e^{-\delta x})$$

for some $\beta > 0$, $\delta > 0$, for each y , as $x \rightarrow \infty$. Similarly, there is a non-zero solution $H_2(x,y)$ to

$$-\alpha H + (A+B + L(g(y)))H_x + \frac{1}{2} H_{xx} + \lambda (H(x+1,y)-H) = 0$$

such that

$$H_2(x,y) = O(e^{\delta x})$$

$$(3.9) \quad H_{2,xx}(x,y) = O(e^{\lambda x})$$

for some $\delta > 0$, $\lambda > 0$, for each y as $x \rightarrow -\infty$.

Then one may write

$$(3.10) \quad V_1(x,y) = J(x,y,A-B) + H_1(x,y)$$

$$(3.11) \quad V_2(x,y) = J(x,y,A+B) + H_2(x,y)$$

and assume that $b(y)$ is determined from (3.6), (3.10), (3.11).

As in Section 2, one then obtains

Theorem 2. Assume that the conditions and results of sections 1,3 hold for $T = \infty$,

The optimal u_1 may be expressed as

$$(3.12) \quad u_1(X_1(t,y),y) = \begin{cases} A-B & \text{if } X_1(t,y) > b(y) \\ A+B & \text{if } X_1(t,y) \leq b(y) \end{cases}$$

where $b(y)$ is obtained from (3.6), (3.10)-(3.11) and

$$(3.13) \quad X_{1t}(t,y) = L(X_1(t,y)) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt}.$$

$$\text{and} \quad X_1(0,y) = g(y) = x.$$

Proof. The proof follows that of Theorem 1.

Let $D = V_{xx}$. From (3.4), (3.5) (omitting (x,y) arguments)

$$(3.14) \quad (A+B + L(g(y)))D_x + \frac{1}{2} D_{xx} - \alpha D - \lambda D = -2 - \lambda D(x+1)$$

where the argument on the left side of (3.14) is x .

From (3.7),

$$(3.15) \quad J_{xx}(x,y,A+B) > 0.$$

Assumption 2, (3.8)-(3.11), (3.15) imply that

$$(3.16) \quad V_{1,xx}(x,y) > 0 \text{ for each } t,y, \text{ as } x \rightarrow \infty$$

and

$$(3.17) \quad V_{2,xx}(x,y) > 0 \text{ for each } t,y, \text{ as } x \rightarrow -\infty.$$

By an argument identical to that given in the proof of Theorem 1, using the appropriate maximum principle ([1], Theorem 18, p. 53), it follows that $D(x,y) \geq 0$ for all (x,y) . Since $V_x(b(y),y) = 0$

by (3.6), it follows that (3.4), (3.5), (3.10), (3.11) constitute a solution to the Bellman equation (3.3). To show that u_1 is optimal, define

$$(3.20) \quad K(X(t,y),y) = V(X(t,y),y)e^{-\alpha t}$$

for $t \geq 0$.

Noting that $K(X(0,y),y) = V(x,y)$, one obtains from the Ito rule ([2], pp. 125-126) for a u and corresponding $X(t,y)$, (omitting $(X(s,y),y)$ arguments on the right side),

$$(3.21) \quad \int_0^t e^{-\alpha s} X^2(s,y) ds + V(X(t,y),y)e^{-\alpha t} - V(x,y) =$$

$$\int_0^t e^{-\alpha s} (X^2(s,y) - \alpha V + \inf_{|u-A| \leq B} (uV_x) + L(g(y))V_x + \frac{1}{2} V_{xx}) ds$$

$$+ \int_0^t e^{-\alpha s} (uV_x - \inf_{|u-A| \leq B} (uV_x)) ds$$

$$+ \int_0^t e^{-\alpha s} (V(X(s,y),s) dN(s)$$

$$+ \int_0^t e^{-\alpha s} V_x dW(s).$$

The second integral on the right of (3.21) is non-negative. Upon taking expectations in (3.21), the fourth term on the right is zero, and combining the first and third terms on the right yields that

$$\begin{aligned}
(3.22) \quad & \int_0^t e^{-\alpha s} E(X^2(s,y)) ds + E(V(X(t,y),y)) e^{-\alpha t} - V(x,y) = \\
& E \int_0^t e^{-\alpha s} (X^2(s,y) - \alpha V + \inf_{|u-A| \leq B} (uV_x) + L(g(y))V_x + \frac{1}{2} V_{xx} \\
& \quad + \lambda(V(X(s)+1,y) - V) ds \\
& + E \int_0^t e^{-\alpha s} (uV_x - \inf_{|u-A| \leq B} (uV_x)) ds.
\end{aligned}$$

By (3.3), the first term on the right of (3.22) is zero.

The second term on the right of (3.22) is non-negative, and is zero for $u = u_1$. By Assumption 2, (3.7)-(3.11), it follows that, for (x,y) fixed, there is a constant $D > 0$ such that

$$(3.23) \quad E(V(X(t,y),y)) e^{-\alpha t} \leq E(|x| + (|A| + B + |L(g(y))|)t + |W(t)| + N(t))^2 e^{-\alpha t} + D e^{-\alpha t}$$

or

$$(3.24) \quad E(V(X(t,y),y)) e^{-\alpha t} \leq C(x,y) t^2 e^{-\alpha t}$$

for some positive constant $C(x,y)$.

Letting $t \rightarrow \infty$ in (3.22), by (3.24) one obtains

$$(3.25) \quad \int_0^\infty e^{-\alpha s} E(X^2(s,y)) ds \geq V(x,y)$$

and

$$(3.26) \quad \int_0^\infty e^{-\alpha s} E(X_1^2(s,y)) ds = V(x,y),$$

so that (3.25), (3.26) imply that u_1 is optimal.

4. Additional Constraints

Certain additional constraints may be incorporated and treated by these methods. An illustrative example in the case $T < \infty$ is the added constraint

$$(4.1) \quad E(X^2(a)) = C$$

where $0 < a \leq T$ and $C > 0$. This may be handled by adding the condition

$$(4.2) \quad v_t(x,t,y)|_{t=a} = e^{-\alpha a} C$$

to conditions (2.5), and proceeding as before. See [3] for another approach.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Optimal Wiener-Poisson control; discounted quadratic cost; Bellman equation; partial differential-difference equations; asymmetric bang-bang control; maximum principle, Ito-rule.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) PLEASE SEE REVERSE SIDE.		

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A one-dimensional Wiener plus independent Poisson control problem with state governed by a partial differential equation has integrated discounted quadratic cost function and asymmetric bounds on the control, which is a function of the current state. A Bellman equation and maximum principle for partial differential equations are used to obtain the optimal closed loop control in bang-bang form. The finite and infinite integral quadratic cost functions are treated separately. ↗

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